Paraxial Theory of Bistable Bright Screening Photovoltaic Solitons in Biased Photovoltaic-Photorefractive Crystals

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Abstract
The propagation characteristics of optical spatial solitons in a biased photovoltaic-photorefractive crystal have been investigated under Wentzzel-Kramers-Brillouin-Jeffreys (WKBJ) approximation. The existence of bi-stable states of the solitons has been identified.

Keywords: soliton, photorefractive crystal, paraxial theory, bi-stability.

1. Introduction
The field of spatial optical solitons in photorefractive crystals has attracted great interest in recent years (Bian et al., 1997; Chen z et al., 2003; Segev et al., 1997; Konar and Asif, 2010; Taya et al., 1996; Zhang et al., 2007; Chen et al., 1997; Konar et al., 2007) since these solitons are promising for all optical beam switching, beam steering, optical interconnections and development of new photonic devices. In addition, since photorefractive crystal PR has the ability to create optical solitons at very low optical power, of order of microwatts (Bian et al., 1997; Chen et al., 2003; Segev et al., 1997; Konar and Asif, 2010). It is the promising and prospective media for experimental verification of theoretical models. Till date, three different types of steady state photorefractive solitons have been predicted. The one which was identified first is the screening solitons. Both bright and dark screening solitons SS in the steady state are possible when an external bias voltage is applied to a non-photovoltaic photorefractive crystal (Chen z et al., 2003; Segev et al., 1997; Konar S and Asif N 2010; Taya et al., 1996; Zhang et al., 2007; Chen et al., 1997). The second kind is the photovoltaic solitons (Konar et al., 2007; Hou et al., 2007) the formation of which, however, requires an unbiased PR crystal that exhibits photovoltaic effect i.e., generation of dc current in a medium illuminated by a light beam. Recently, a third kind of photorefractive solitons has been introduced (Jinsong, 2003) which arises when an electric field is applied to a
photovoltaic photorefractive crystal. These solitons owe their existence to both photovoltaic effect and spatially non-uniform screening of the applied field and are also known as screening photovoltaic solitons SP. Both bright and dark SP solitons have been investigated. It has been shown that if the bias field is much stronger than the photovoltaic field, then the screening photovoltaic solitons are just like screening solitons. On the other hand, if the applied field is absent, then they degenerate into photovoltaic solitons in the closed circuit condition. In the present communication, we reveal existence of two-state property of these solitons. The existence of this two-state property is due the non Kerr index change which governs the dynamics of these solitons.

2. Mathematical model

We proceed to investigate the characteristic of optical spatial solitons which are propagating in a biased photovoltaic-photorefractive crystal along the z- axis and is permitted to diffract only along the x-direction. The optical beam is linearly polarized along the x- axis which is also the direction of applied external electric field. To describe spatial solitons in such circumstances, we begin with the following evolution equation

\[ \frac{i}{\varepsilon} \frac{\partial U}{\partial \xi} + \frac{1}{2} \frac{\partial^2 U}{\partial S^2} - \beta \frac{U}{1 + |U|^2} + \alpha \frac{|U|^2 U}{1 + |U|^2} = 0 \]  

(1)

Where \( A \) is the normalized envelope of the optical beam, \( \beta = \left( k_0 x_0 \right)^2 \frac{n_e^4 r_{\text{eff}}}{2} E_0 \), 

\[ \alpha = \left( k_0 x_0 \right)^2 \frac{n_e^4 r_{\text{eff}} E_p}{2} \], 

\[ E_p = \frac{k p \gamma R N_A}{e \mu}, \quad k_0 = \frac{2 \pi}{\lambda_0}, \quad s = \frac{x}{x_0}, \quad \xi = \frac{z}{k_0 n_e x_0^2}, \]

\( n_e \) is the unperturbed extra ordinary index of refraction, \( \lambda_0 \) is the free space wave length of the light employed, \( r_{\text{eff}} \) is the effective electro-optic coefficient, \( E_0 \) is the external bias electric field, \( E_p \) is photovoltaic field constant, \( x_0 \) is an arbitrary spatial width; \( \gamma, \mu, e, k_p \) and \( N_A \) are respectively the carrier recombination rate, electron mobility, electronic charge, photovoltaic constant and the acceptor density. In order to investigate the existence and stability of bright spatial solitons, we solve the beam evolution equation taking the following ansatz:

\[ U(\xi, s) = U_0 (\xi, s) e^{-i \Omega(\xi, s)} \]  

(2)

By virtue of the above ansatz, equation 1 yields the following two equations:

\[ \frac{\partial U_0}{\partial \xi} \cdot \frac{\partial U_0}{\partial S} + \frac{\partial \Omega}{\partial S} \cdot \frac{\partial U_0}{\partial S} - \frac{1}{2} U_0 \frac{\partial^2 \Omega}{\partial S^2} = 0 \]  

(3)

and

\[ U_0 \frac{\partial \Omega}{\partial \xi} \cdot \frac{\partial^2 U_0}{\partial S^2} - \frac{1}{2} U_0 \left( \frac{\partial \Omega}{\partial S} \right)^2 - \beta \Phi_1 (\xi, s) U_0 + \alpha \Phi_2 (\xi, s) U_0 = 0 \]  

(4)
Where \( \Phi_1(\xi, s) = \frac{1}{1 + |U_0|^2} \) and \( \Phi_2(\xi, s) = \frac{|U_0|^2}{1 + |U_0|^2} \) (5)

In equation 4, \( \Phi_1(\xi, s) \) and \( \Phi_2(\xi, s) \) account for the non linearity induced in the PR material due to the space charge induced refractive index change by the external bias field and the photovoltaic field respectively. The last two terms control the diffraction of the beam, leading to its shape preserving propagation. We look for a self-similar spatial soliton solution for which the electromagnetic field energy is confined in the central region of the beam. Of the many possible solutions, the Gaussian solution gives very good analytical results, comparable to those found from pure numerical simulations.

Hence, the amplitude and phase of solitons are taken in the following form:

\[
U_{00}(\xi, s) = \frac{U_{00}}{\sqrt{f(\xi)}} e^{-s^2/2r^2 f^2(\xi)} \tag{6a}
\]

\[
\Omega(\xi, s) = \frac{s^2}{2} \Gamma(\xi) + \Psi(\xi) \tag{6b}
\]

\[
\Gamma(\xi) = -\frac{1}{f(\xi)} \frac{df(\xi)}{d\xi} \tag{6c}
\]

where \( U_{00} \) is the square root of the normalized peak power of the soliton, \( r \) is a positive constant and \( f(\xi) \) is the variable beam width parameter such that the product \( rf(\xi) \) gives the spatial width of the soliton. The nonlinear contributions \( \phi_1 \) and \( \phi_2 \) to the refractive index are expanded in Taylor series, from which under first order approximation we obtain

\[
\Phi_1(\xi, s) \approx \frac{1}{1 + \frac{U_{00}^2}{f(\xi)}} + s^2 \frac{r^2 f^3(\xi)}{\left(1 + \frac{U_{00}^2}{f(\xi)}\right)^2} \tag{7}
\]

\[
\Phi_2(\xi, s) \approx \frac{U_{00}^2}{f(\xi)} + s^2 \frac{U_{00}^2}{\left(1 + \frac{U_{00}^2}{f(\xi)}\right)^2} \tag{8}
\]

Equations 6-8 upon substitution into 4 result in an identity wherein we can equate the coefficients of various powers of \( s^2 \) on both sides of the identity. We thus obtain the evolution equation for the beam width parameter \( f(\xi) \) as
\[
\frac{d^2 f(\xi)}{d\xi^2} = \frac{1}{r^4 f^3(\xi)} - 2(\alpha + \beta) \frac{P_0}{r^2 f^2(\xi)} \left\{ 1 + \frac{p_0}{f(\xi)} \right\}^2
\]

(9)

The above equation describes the evolution of the beam width parameter of the spatial soliton of power \( P_0 = U_0^2 \) in photo voltaic photorefractive crystal. Depending on the value of power and the relative strength of the system parameters \( \alpha \) and \( \beta \), the optical solitary wave may diverge, compress or be trapped. A trapped solitary wave i.e. a spatial soliton, is achieved when the beam width remains invariant so that the left-hand side of equation 9 is zero. This equation results in the following quadratic equilibrium equation giving the threshold power \( p_{0th} \) for stationary propagation as

\[
\frac{1}{r^4} = \frac{2(\alpha + \beta)P_{oth}}{r^2 (1 + p_{0th})^2}
\]

(10)

Where \( p_{0th} \) represents the threshold value of power \( p_0 \). The variation of \( r \) versus the threshold power \( p_{0th} \) has been depicted in figure 1.

![Figure 1](image-url)

Figure 1. Variation of equilibrium spatial width \( r \) with threshold peak power \( P_{0th} \) of the spatial soliton. \( \alpha = 31.58, \beta = 157.9 \).

The figure clearly demonstrates the existence of bi-stable soliton states, i.e. solitons of a particular width can be formed inside the crystal at two threshold powers, \( P_{0th1} \) and \( P_{0th2} \). The threshold power for stationary propagation can be divided into low and high power regimes. Variation of soliton width with threshold power is more pronounced in the low power than the high power regime. In order to examine the behaviour of the spatial soliton at different
peak powers, we have presented the variation of $f$ with normalized distance of propagation in figure 2.

![Graph showing variation of variable beam width parameter $f(\xi)$ with normalized distance of propagation $\xi$, at four different soliton peak powers. $\alpha = 31.58, \beta = 157.9$.](image)

Figure 2: variation of variable beam width parameter $f(\xi)$ with normalized distance of propagation $\xi$, at four different soliton peak powers. $\alpha = 31.58, \beta = 157.9$.

For demonstration, four different values of power, $p_1, p_2, p_3$ and $p_4$, have been chosen. Particularly, $p_1 = 0.09999$, $p_2 = p_{\text{nth}} = 3.390$, $p_3 = p_{\text{th}} = 33.90$ i.e. $p_1 < p_{\text{nth}} < p_3 < p_{\text{th}}$ and $p_3 < p_{\text{th}}$. It is evident that at $p_1$, the beam width diverges to a very large since the soliton peak power is less than the threshold power. At threshold power $p = p_{\text{th}}$, the spatial soliton propagates maintaining its shape unchanged, which has been manifested by constant beam width $f = 1$. The beam width of a spatial soliton oscillates with oscillation amplitude less than unity when the soliton peak power lies between two threshold values $p_{\text{nth}}$ and $p_{\text{th}}$.

3. Conclusion

In this paper, we have investigated the propagation characteristics of spatial solitons in photovoltaic-photorefractive crystals. We have identified for different propagation regimes. This also reveals the existence of two state properties of solitons.

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References:

Konar S and Asif N 2010, External field- controlled photovoltaic symbiotic spatial soliton pairs in two-photon photorefractive crystal”Phys,Scr.811 015401
Zhang Y, Hou C-F and Dong S-X 2007, grey spatial soliton due to two photon photorefractive effect” Chin.Phys.16 0159