Upward Laminar Flow Around A Circular Cylinder Using Nanofluids

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ABSTRACT
Analysis of fluid flow and heat transfer around bluff obstacles such as circular cylinders at low Reynolds numbers has been a subject of intense research for several decades. When flow passes over a bluff body, wakes from behind it as a result of flow separation and formation of recirculation zones. The flow structure strongly depends on the shape and size of the body, the inflow and outflow conditions and the blockage parameter. The study examines the effect of aiding buoyancy on two-dimensional laminar flow across a circular cylinder using different types of nanofluids. It is numerically investigated using a finite volume technique. The governing equations were solved using Computational Fluid Dynamics (CFD) with the aid of SIMPLEC algorithm. The influence of aiding buoyancy is studied for the range of parameters $0.5 \leq Ri \leq 0.5$, $50 \leq Re \leq 150$, and the blockage ratios of $B=0.02$ and $0.25$. The investigation also covers different volume fractions of nanoparticles in range of $1\% \leq \phi \leq 10\%$. The nanoparticles diameter is also varied in the range of $25\text{nm} \leq dp \leq 60\text{nm}$. Besides that, various types of nanoparticles are used, such as $\text{Al}_2\text{O}_3$, $\text{CuO}$, $\text{SiO}_2$, and $\text{TiO}_2$. The results show that $\text{Al}_2\text{O}_3$ nanofluid has the highest Nusselt number when compared to other type of nanofluids. In case for nanoparticle concentration, $0.01$ volume fraction of nanoparticles has the highest Nusselt number. Besides, lower nanoparticles diameter ($dp=25\text{nm}$) yields higher Nusselt number. It is inferred that nanoparticles with smaller diameters increase the uniformity of the particle distribution around the circular cylinder.

Keywords: Fluid flow, Heat transfer, Reynold numbers, aiding buoyancy, blockage ratios, and nanofluids
1. Introduction

Analysis of fluid flow and heat transfer around bluff obstacles such as circular cylinders at low Reynolds numbers has been a subject of intense research for several decades. The transport processes occurring behind the bluff bodies have tremendous engineering importance since these are often found crucial in numerous technological applications such as heat exchangers, solar heating systems, natural circulation boilers, nuclear reactors, dry cooling towers, electronic cooling, and so on. When flow passes over a bluff body, wakes from behind it as a result of flow separation and formation of recirculation zones. The flow structure strongly depends on the shape and size of the body, the inflow and outflow conditions and the blockage parameter. The flow becomes more complicated when the wake is further influenced by heat transfer. It should be recognized that for low to moderate Re flows, the buoyancy effect can significantly influence the flow field, thereby affecting heat transfer characteristics. When the flow velocity is not very high but the temperature difference between the body and the fluid is significantly high, the heat transfer behavior is strongly influenced by thermal buoyancy. The thermal buoyancy plays a role of paramount importance on estimating the wake behavior since the wake structure is perturbed due to the superimposed thermal buoyancy resulting in hydrodynamic instabilities even at small Re. The wake behavior is significantly different for the case of cross flow over a horizontal cylinder in comparison to the flow in a vertical configuration under the influence of buoyancy. Buoyancy forces usually enhance the surface heat transfer rate when they aid the forced flow. For the aiding situation (flow past a heated body), the forced flow is in the same direction as the buoyancy force.

In the context of laminar mixed convection heat transfer around circular cylinder subjected to vertically upward/downward flow, Oosthuizen and Madan, (1971) studied experimentally the unsteady mixed convection for Re=100-300. Merkin (1977) pointed out that heating the circular cylinder delays separation and finally, the boundary layer does not separate at all. Jain and Lohar (1979) reported an increase in shedding frequency with an increase in cylinder temperature. Farouk and Guceri (1982) investigated numerically the laminar natural channel walls in the steady flow regime. Badr (1984) and Badr (1985) studied the laminar mixed convection heat transfer from an isothermal horizontal circular cylinder for the two cases when the forced flow is directed either vertically upward (parallel flow) or downward (contra flow). The buoyancy-aided (0≤Ri≤4) steady convection heat transfer at low Re (20, 40 and 60) from a horizontal circular cylinder in a vertical adiabatic duct has been studied numerically by Ho et al. (1990) for the blockage parameters of B=0, 0.1667, 0.25 and 0.5. They observed a significant enhancement of the pure forced convection heat transfer due to the blockage effect. However, there was appreciable degradation of the buoyancy-aided enhancement in the heat transfer rate. Chang and Sa (1990) investigated the phenomenon of vortex shedding from a heated/cooled circular cylinder in the mixed convection regime and predicted the degeneration of purely periodic flows into a steady vortex pattern at a critical Richardson number of 0.15. Nakabe et al. (1996) studied the effect of buoyancy on the channel-confined (channel walls maintained at ambient
temperature) flow across a heated/cooled circular cylinder with parabolic inlet velocity profile, by a finite difference method. Three flow configurations were considered: (i) aiding buoyancy at \( \text{Re}=80 \) and 120, \( 0 \leq \text{Ri} \leq 1.6 \) and \( B=0.15 \) and 0.3 (ii) opposing buoyancy at \( \text{Re}=50 \), \( -1 \leq \text{Ri} \leq 0 \) and \( B=0.15 \) and (iii) cross-stream buoyancy at \( \text{Re}=80 \) and \( B=0.3 \). For the aiding buoyancy case, the authors found three main results: first, at constant \( \text{Re} \), the value of \( \text{Ri} \) at which the vortex shedding degenerates (into twin vortices) decreases with the increasing \( B \); second, at constant \( B \), the value of \( \text{Ri} \) at which vortex shedding degenerates increases with \( \text{Re} \); and third, at constant \( \text{Ri} \), the value of \( \text{Re} \) at which vortex shedding starts increases with increasing \( B \).

By exploiting a finite element method, Singh et al. (1998) determined the flow field and temperature distribution around a heated/cooled circular cylinder placed in an insulated vertical channel, with parabolic inlet velocity profile at \( \text{Re}=100 \), \( -1 \leq \text{Ri} \leq 1 \) and \( B=0.25 \). They demonstrated that in the viscous region, at the vicinity of the aft of the cylinder, the inertia force is opposed by the force due to an adverse pressure gradient, the negative buoyancy force and viscous force. Furthermore, they observed that the vortex shedding stopped completely at a critical Richardson number of 0.15, below which the shedding of vortices into the stream was quite prominent. The effect of buoyancy on the flow structure and heat transfer characteristics of an isolated square cylinder in upward flow was investigated numerically by Sharma and Eswaran (2004) for \( \text{Re}=100 \) and \( \text{Pr}=0.7 \). Like the circular cylinder case, the degeneration of the Karman vortex street was also observed to occur at a Richardson number of 0.15 from their study for the bluff square cylinder. In another article, Sharma and Eswaran (2005) studied the effect of channel-confinement of various degree (\( B=0.1, 0.3 \) and 0.5) on the upward flow and heat transfer characteristics around a heated/cooled square cylinder by considering the effect of aiding/opposing buoyancy at \( -1 \leq \text{Ri} \leq 1 \), for \( \text{Re}=100 \) and \( \text{Pr}=0.7 \). They observed that with an increase in the blockage parameter, the minimum heating (critical \( \text{Ri} \)) required for the suppression of vortex shedding decreases up to a certain blockage parameter (\( B=0.3 \)), but thereafter increases. Singh et al. (2007) performed a comprehensive schlieren-interferometric study for the wakes behind heated circular and square cylinders placed in a vertical test cell. A detailed dynamical characteristic of vertical structures was reported in their study. The problem of the laminar upward mixed convection heat transfer for thermally developing air flow in the entrance region of a vertical circular cylinder under buoyancy effect and wall heat flux boundary condition has been numerically investigated by Hussein and Yasin (2008) through an implicit finite difference method and the Gauss elimination technique. The investigation covers Reynolds number range from 400 to 1600 and the heat flux from 70W/m\(^2\) to 400W/m\(^2\). The results revealed that the secondary flow created by natural convection have a significant effect on the heat transfer process.

2. Problem Description

The geometry of the problem considered in this study consists of a fixed two-dimensional circular cylinder of diameter \( d \) heated or cooled to a constant temperature \( T_w \) and exposed to an upward stream of temperature \( T_\infty \). For the unconfined (\( B=\frac{d}{H}=0.02 \), where \( H \) is the width of the
computational domain) and channel confined (B=0.25) flows. Two adiabatic vertical walls of finite length are placed at a distance of H/2 on either side of the center of the cylinder for the channel confined flow cases, whereas, artificial confining boundaries are considered for the unconfined flow case. The center of the cylinder is placed on the vertical axis at a downstream distance of $L_u=8d$ from the entry plane. The lengths of the computational domains are fixed as $L=L_u+L_d=24d$ for $B=0.25$ and $L=34d$ for $B=0.02$. These values are chosen as so to reduce the effect of the inlet and outlet boundary conditions on the flow patterns in the vicinity of the obstacle. Furthermore, these choices are also consistent with the other contemporary studies available in the literature.

### Fig. 1 Schematic diagram of the computational domain (a) B=0.02 and (b) B=0.25

#### 3. Governing Equations

The dimensionless governing equations for this two-dimensional, laminar, incompressible flow with constant thermo-physical properties along with Boussinesq approximation and negligible dissipation effect can be expressed in the following forms:
3.1 Continuity Equation

\[ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0 \]  

(1)

\( \partial U \) and \( \partial V \) are change of interstitial velocity component at \( x \) and \( y \) direction while \( \partial X \) is distance in horizontal axis from reference point and \( \partial Y \) is distance in vertical axis from the reference point.

3.2 Momentum Equation

\[ \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]

\[ \frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Ri \theta \]  

(2)

3.3 Energy Equation

\[ \frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \frac{1}{Pe} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \]  

(3)

where \( U, V \) are the dimensionless velocity components along \( X \) and \( Y \) directions of a Cartesian coordinate system, respectively; \( P \) and \( \tau \) are the dimensionless pressure and time; \( Re (=\nu \times d/\nu) \) is the Reynolds number based on the cylinder diameter with \( \nu \) being the average velocity at the inlet; \( Ri (=Gr/Re^2) \) is the Richardson number; \( Gr (=g\beta(T_w-T_\infty)d^3/\nu^2) \) is the Grashof number with \( g \) and \( \beta \) being the gravitational acceleration and volumetric expansion coefficient; \( \theta \) is the dimensionless temperature; and \( Pe (=RePr) \) is the Pe’clet number with \( Pr (=\nu/\alpha) \) being the Prandtl number. The fluid properties are described by the density \( \rho \), kinematics viscosity \( \nu \), and thermal diffusivity \( \alpha \). The dimensionless variables are defined as follows:

\[ U = \frac{u}{u_\infty}, V = \frac{v}{v_\infty}, X = \frac{x}{d}, Y = \frac{y}{d}, P = \frac{p}{\rho u_\infty^2}, T = \frac{T - T_\infty}{T_w - T_\infty}, \tau = \frac{\nu \tau_\infty}{d} \]  

(4)

The corresponding dimensional quantities are denoted by \( u, v, x, y, p, T, \) and \( t \), respectively.
4. Boundary Conditions

A parabolic velocity profile with the average velocity \( \nu_{av} \) is considered at the inlet for the channel confined flow; whereas, a uniform free stream with velocity, \( \nu_1 \) is assumed for the unconfined flow. The vertical channel walls are assumed insulated, and far-field boundary conditions are imposed on the artificial confining boundaries. The cylinder temperature \( T_W \) is governed by the magnitude of the Richardson number. A no-slip boundary condition is imposed on both the cylinder and channel walls. The exit boundary is located sufficiently far downstream from the region of interest; hence, an outflow boundary condition is proposed at the outlet. Mathematically, one can write the following.

At the inlet:

\[
U = \theta = 0, \quad V = \begin{cases} 
1.5 \left(1 - \frac{x^2}{4}\right) & \text{for } B = 0.25 \\
1 & \text{for } B = 0.02
\end{cases}
\]

At the outlet:

\[
\frac{\partial U}{\partial Y} = \frac{\partial V}{\partial Y} = \frac{\partial \theta}{\partial Y} = 0
\]

At the vertical boundaries:

\[
U = V = \frac{\partial \theta}{\partial Y} = 0 \quad \text{for } B = 0.25 \\
U = \theta = 0, \quad V = 1 \quad \text{for } B = 0.02
\]

At the cylinder surface:

\[
U = V = 0, \quad \theta = 1
\]

The flow is assumed to start impulsively from rest.

5. Method Of Solution

The computational fluid dynamic FLUENT 6.3.26 was used for solving the problem under consideration. The conservation equations subjected to the aforementioned boundary conditions
are solved using a finite-volume-based method according to the SIMPLEC algorithm in a collocated grid system. A second order upwind scheme for discretizing the convective terms, and a central difference scheme for the diffusive terms of the momentum and energy equations are used. The time discretization is carried out by a second order accurate fully implicit Adams-Bashforth scheme. The conditions necessary to prevent numerical oscillations are determined from the Courant-Friedrichs-Lewy (CFL) and the grid Fourier number criteria. The final time step size is usually taken as the minimum of the two criteria mentioned above. Furthermore, the time step size has been varied from 0.01 to 0.1 to determine an optimum value that results in less computational time, but produces sufficiently accurate results. A dimensionless time step size of 0.05 is finally used in the computations satisfying all of the above restrictions. A body-force-weighted pressure interpolation technique is used to interpolate the face pressure from the cell center value. The discretized governing equations are finally solved by an algebraic multigrid solver developed by the same research group earlier for such flows (Chatterjee et al., 2010). The convergence criteria for the inner (time step) iterations are set as $10^3$ for all discretized governing equations.

6. Results And Discussion

6.1 The effect of geometry and Richardson number

In this numerical simulation, the effect of geometry and Richardson number were investigated. Geometry was changed from blockage ratio 0.02 to 0.25 and the Richardson number was changed from 0.1 to 0.5. The nanofluid used is only the titanium dioxide-water with volume fraction of 10%. The result was monitored three various views which are the change in Strouhal number, Critical Richardson number and Nusselt number.

6.2 The effect of Blockage and Reynolds number on Strouhal number

The first output to be investigated is Strouhal number. To further establish the observations regarding the suppression of vortices at the critical Ri for different Re, the dimensionless frequency of vortex shedding (Strouhal number) is plotted against Ri for the two blockage parameters in figure 5. The Strouhal number (St) is obtained from the fast Fourier transform (FFT) of the transverse velocity component signal at the location 3d downstream of the cylinder on the channel centerline. The Strouhal number is found to decrease with increased cooling of the cylinder below the fluid temperature (i.e., with decreasing Ri) for all Re. This can be attributed to the fact that with an increased cooling of the cylinder, the fluid in the vicinity of the cylinder becomes denser and, hence, the vortices stay attached to the cylinder for long periods. Up to the critical values of Ri, St increases steadily; however, for Ri higher than the critical values, the frequency becomes suddenly zero, i.e., the shedding has stopped completely as a result of the breakdown of the Karman vortex street. It should be mentioned that for higher blockage ratio, the shedding frequency is observed to be more compared to that for the lower one, particularly for lower values of Ri, as a result of the effect of the confining walls. At low
Re, the vorticity pumped into the wake from the boundary layers in the cylinder can be diffused away from the shear layers merely by viscous diffusion. However, as the flow Re number increases, viscous diffusion alone cannot keep up with the increase vorticity production in the upstream boundary layers and vortices break away at the regular intervals, constituting vortex shedding. Hence, the flow destabilizes at high Re and accordingly more heating is required to weaken the breakdown of Karman Vortex Street.

**Fig. 2** Variation of Strouhal number with Richardson number

### 6.3 The effect of blockage parameters on Critical Richardson number

The second output to be investigated is critical Richardson number. The critical Ri is not a universal property for the degeneration of the vortex shedding mechanism, and is a function of the flow Reynolds number (Jain and Lohar, 1979). Figure 3 depicts the dependence of the critical Richardson number on the Reynolds number for the two blockage ratios. The critical Ri is found to increase with Re at the constant blockage ratio. At low Reynolds number, the vorticity pumped into the wake from the boundary layers on the cylinder can be diffused away from the shear layers merely by viscous diffusion (Farouk and Gucerı, 1982). However, as the flow Reynolds number increases viscous diffusion alone cannot keep up with the increased vorticity production in the upstream boundary layers, and vortices break away at regular intervals, constituting vortex shedding. Hence, the flow destabilizes at higher Re and, accordingly, more heating is required to weaken the breakdown of the Karman vortex street. Figure 6 further reveals that the critical Ri is always higher for the lower blockage ratio at a particular Re. This is due to the increase in the stability of the wake for the channel confined case (higher blockage), which requires less heating for stabilization of the flow.
6.4 The effect of blockage and Reynolds number on Nusselt number

The third output to be investigated is Nusselt Number. Grid size $110 \times 312$ is used for blockage ratio 0.25 and grid size $220 \times 312$ is used for blockage ratio 0.02 to run the simulations. From the simulation, it can be seen from figure 7, that the nusselt number is found to increase with Re as usual for both the two blockage ratios. However, the variation of Nu to increase with Ri has some interesting features. For example, below the critical Ri, increasing magnitude of Ri serves to increase Nu in general but with a lower gradient, whereas, above the critical Ri, it increases rapidly. This is because of the fact that with the sudden stopping of shedding at the critical Ri, the vortices become smaller in size resulting in a reduction of the poor heat transfer zones and consequently the heat transfer is enhanced. It is observed from figure 4 that the higher blockage blockage ratio ($B=0.25$) yields higher Nu, since the higher blockage increases the shedding frequency and hence the vortices are shed at a faster rate resulting in an enhanced heat transfer rate.
6.5 The effect of Reynolds number and volume fraction on Nusselt number

Recirculation region around the circular cylinder affected the heat transfer coefficient and causing it to increase with volume fraction of nanofluids. From the results below, higher volume fraction yields higher nusselt number for both blockage thus to enhance heat transfer coefficient. Besides that, higher Reynolds number also yield higher nusselt number for both blockage as can be seen in figure 5 and figure 6.

Fig. 4 Variation of Nusselt number with Richardson number

Fig. 5 Variation of Nusselt number with Richardson number for B=0.25
6.6 The effect of nanoparticle diameter on Nusselt number

Nanoparticles with smaller diameter increases the uniformity of the particle distribution around the circular cylinder. Therefore, lower nanoparticle diameter yields higher nusselt number as can be seen in figure 7 and figure 8.
6.7 The effect of nanofluids on Strouhal number

From the results below, vortex shedding frequency increases with increases heating and suddenly reduce to zero at the critical Ri number. TiO₂ has higher Strouhal number and higher critical Ri number if compared to other nanofluids as can be seen in figure 9 and figure 10. Besides that, higher blockage parameter has lower critical Ri number. This is due to the different physical properties (such as density, capacity, thermal conductivity etc) at high different temperature which affect the flow rate and Nusselt number.
6.8 The effect of nanofluids on Nusselt number

From the results below (figure 11 and figure 12), TiO$_2$ yields the highest Nusselt number. Meanwhile, pure water has the lowest nusselt number. This is because pure water has the highest thermal conduction if compared to nanofluids.
7. Conclusion

The paper was divided into two phases, which are searching the information of heat transfer augmentation percentage for mixed convection flow around a circular cylinder using nanofluids. The second phase consists of simulation of the effect of different parameters on heat transfer characteristic around the circular cylinder using the finite volume based on SIMPLEC method. This paper was concentrated on Nusselt number and Strouhal number using nanofluid around a circular cylinder. The effect of combining different nanofluids of different Richardson numbers, different nanoparticle diameters and different nanoparticle concentrations in laminar flow through 2-dimensional model is numerically investigated.

The result obtained the Nusselt number increases due to the increase of Reynolds number and Titanium Dioxide has the highest Nusselt number. Besides that, higher blockage ratio has lower critical Richardson number but the critical Richardson number is found to increase with Reynolds number. In this case, it obvious that geometry plays a big role in heat transfer coefficient using nanofluids. In case for nanoparticle concentration, $\phi=10\%$ has the highest Nusselt number. It is inferred that nanofluids with low thermal conductivity nanoparticles have higher Nusselt number than the high thermal conductivity nanoparticles. Recirculation region around the circular cylinder affected the heat transfer coefficient at the trailing edge and causing it to increase with the volume fraction of nanoparticles.

The following conclusions can be made from the results:

- $\text{Al}_2\text{O}_3$ has the highest Nusselt number when compared to other type of nanofluids.
The Nusselt number increases due to increase of Reynolds number.

The critical Richardson number is found to increase with Reynolds number.

The critical Richardson number is found to decrease with blockage ratio.

Nanoparticle at $\varphi=10\%$ has the highest Nusselt number.

The Nusselt number is found to be higher for higher blockage.

Lower nanoparticle diameter yields higher Nusselt number.

References


