Non newtonian fluid flow under the effect of chemical reaction and Ion and Hall currents over a moving cylinder

Khaled K. Jaber and Faris M. Al-Athari

Department of Mathematics,
Faculty of Science and Information Technology, Zarqa University

Khaledjaber4@yahoo.com

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ABSTRACT
Laminar forced convection boundary layer flow of an electrically conducting non-Newtonian power-law fluid over a continuously Moving isothermal cylinder subject to normal and uniform strong transverse magnetic field is investigated, taking into account the effects of Hall, ion-slip currents and Ohmic heating.

Approach: Appropriate transformation are employed to transform the governing partial differential equations into non-linear ordinary coupled differential equations, which have been solved numerically using the fourth-order Runge-Kutta scheme with the shooting method.

Result: Numerical results illustrating the effects of all involved parameters on the velocity, temperature and concentration profiles, the friction coefficient, Nusselt and Sherwood numbers are presented and discussed.

Keywords: generalized Schmidt number, Eckert number, Hall current, Ion-Slip current, Ohmic heating, non-Newtonian power-law fluid.
Nomenclature

\( f \) Dimensionless stream function
\( r \) Direction normal to the cylinder’s

Greek Symbols

\( \alpha \) Thermal diffusivity
\( \beta \)
\( \eta \) similar variable
\( \gamma \) Reaction order

Grashof number
\( \nu \) Kinematical viscosity

\( \theta \) Dimensionless temperature
\( \phi \) Dimensionless concentration

Consistency coefficient

Density

Concentration

Transverse curvature parameter

Generalized Schmidt number

Skin friction

Dimensionless streamwise

Stream function

Subscripts

Property at the wall

Freestream condition

Superscripts

\( \delta \) Reaction rate parameter

\( n \) Power law viscosity index

\( U \) Cylinder velocity

Differentiation with respect to

1. Introduction

This paper considers laminar, magnetohydrodynamic flow of a non-Newtonian fluid over an isothermal moving cylinder. This type of fluid is of fundamental importance due to the increasing number of its applications in various engineering systems. Wire or fiber coating, foodstuff processing and transpiration cooling are examples of practical applications of such systems. The problem of laminar flow of a Newtonian fluid on a cylinder moving with constant speed was initially studied by Sakiadis (1961). Later, this problem has been investigated under various physical conditions by Rotte and Beek (1968); Bourne and Elliston (1970); Gampert and Angew (1974); Vlegglaar (1977); Kranis and Pechoc (1978) and Ganesan and Loganathan (2002). Literature on MHD convective heat transfer, keeping in mind the effects of Hall and ion-slip currents is very extensive due to its technical importance in many industrial applications. Some literature surveys and reviews of pertinent work in this field are documented by Pop and Watanabe (1994) and Abo-Eldahab et al. (2000). All the previous works on convective flow have focused on the flow of Newtonian fluids. On the other hand, the flow of non-Newtonian fluids has not received as much attention. In fact, in recent years,
non-Newtonian fluids have been appearing in increasing number. Pop et al. (1990) considered the steady laminar forced convective boundary layer of power–law non-Newtonian fluids on a continuously moving cylinder with the surface maintained at a uniform temperature or a uniform heat flux. Wang (1996) studied the steady laminar forced convection heat transfer from a moving or stationary cylinder to a quiescent or flowing non-Newtonian fluid. Agarwal et al. (2002) investigated the laminar momentum and thermal boundary layers of power-law fluids over a slender cylinder. Abo-Eldahad and Salem (2005) extended the previous work on power-law non-Newtonian fluids by considering the heat and mass transfer over a continuously moving cylinder in the presence of a uniform strong magnetic filed. The effect of mass transfer on the flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction were studied by Das et al. (1994). Based on the above brief review, it is obvious that the Hall and ion-slip currents effects and viscous dissipation on MHD flow of power-law non-Newtonian fluids have not been investigated yet. Therefore, the aim of the present work is to drop the neglecting of Ohmic heating and examine the effect of Hall and ion-slip current on the steady, laminar and incompressible MHD flow of a power-law non-Newtonian fluid in the presence of a strong uniform magnetic field. The heat transfer and mass diffusion chemical species with first and higher order reactions are considered. The temperature and concentration differences between the cylinder surface and the ambient fluid were assumed to vary as a power-law function of the distance along the cylinder. It is assumed that the magnetic Reynolds number is small enough to neglect the induced magnetic field. The fluid properties are assumed to be constant. Also it is assumed that the viscous dissipation and the heat generated by the chemical reaction are negligible. The boundary layer equations are transformed into a system of non-linear ordinary coupled differential equations which is solved by employing the fourth order Runge-Kutta scheme with the shooting method.

2. Mathematical Formulation

Consider the laminar, steady boundary layer flow of an electrically conducting power-law non-Newtonian Fluid over a continuously axially moving cylinder. The x-axis is taken along the axis of the cylinder, and r-axis is taken normal to it. It is assumed that the surface of the cylinder is maintained at a power-law temperature and concentration, also the chemical reaction source or sink term is retained in the concentration equation. The flow is subjected to a uniform strong magnetic filed of strength B₀, which acts in the positive r-axis direction, so the Hall and ion-slip currents significantly affect the flow. The effect of the Hall current give rise to a force in the z-direction, which induces across flow in that direction and hence the flow becomes three-dimensional. Also Ohmic heating is considered to explore the effect of magnetic filed on the thermal transport in the boundary layer. The fluid properties are assumed to be constant. Also viscous dissipation and the induced magnetic field are negligible.

The equation of electric charge conservation \( \nabla \cdot \mathbf{J} = 0 \) along with the fact that the plate is electrically non–conducing gives that \( j_y = 0 \) everywhere in the flow. In the absence of electric filed the current density components \( J_x \) and \( J_z \) as obtained from the generalized Ohm’s law are given by:
\[ J_x = \frac{\sigma B_0}{\alpha^2 + \beta^2_0} (\beta_e u - \alpha_e w) \]  

(1)

\[ J_z = \frac{\sigma B_0}{\alpha^2_e + \beta^2_e} (\alpha_e u + \beta_e w) \]  

(2)

Where \( \alpha_e = 1 + \beta_i \beta_e \), \( \beta_e \) is the Hall parameter, \( \beta_i \) is the ion-slip parameter and \( \sigma \) is the electrical conductivity.

Under the above assumptions, for non–Newtonian fluid the steady boundary layer equations are

\[ \frac{\partial}{\partial x} (ru) + \frac{\partial}{\partial r} (rv) = 0 \]  

(3)

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{K}{\rho \alpha} \left[ \left| \frac{\partial u}{\partial r} \right|^{n-1} \frac{\partial u}{\partial r} \right] - \frac{B_0}{\rho} J_z \]  

(4)

\[ u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial r} = \frac{K}{\rho \alpha} \left[ \left| \frac{\partial w}{\partial r} \right|^{n-1} \frac{\partial w}{\partial r} \right] + \frac{B_0}{\rho} J_x \]  

(5)

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{\alpha}{r} \left[ r \frac{\partial T}{\partial r} \right] + \frac{\sigma B_0^2}{\rho \alpha} (u^2 + w^2) \]  

(6)

\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial r} = \frac{D}{r} \left[ r \frac{\partial C}{\partial r} \right] - k(C - C_w)^\gamma \]  

(7)

The boundary conditions corresponding to equations (3) – (7) are

\[ u = U, \quad v = 0, \quad w = 0, \quad T = T_w (= T_\infty + Ax^1) \quad C = C_w (= C_\infty + Bx^m) \quad \text{at} \ r = r_0 \]

\[ u \to 0, \quad v \to 0, \quad w \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \ r \to \infty. \]  

(8)

where \( u, v \) and \( w \) are the velocity components along \( x, r \) and \( z \) directions, respectively, \( n \) is the power law viscosity index, \( K \) is the consistency coefficient, \( \rho \) is the density, \( \sigma \) is the electrical conductivity of the fluid, \( T \) is the temperature of the fluid, \( C \) is the concentration of the species of the fluid, \( D \) is the diffusion coefficient, \( k \) is the reaction rate constant, \( \gamma \) is the...
reaction order, \( r_0 \) is the radius of the cylinder, \( U \) is the cylinder velocity, \( T_\infty \) and \( C_\infty \) are the free stream temperature and concentration respectively, \( \alpha \) is the thermal diffusivity and \( A, B, m \) and \( \lambda \) are prescribed constants. In order to simplify the mathematical analysis the following transformation is introduced:

\[ \xi = \frac{x}{r_0}, \quad \eta = \frac{r^2 - r_0^2}{2r_0x} \text{Re}_x^{\frac{1}{n+1}}, \quad f(\xi, \eta) = \left( \frac{\text{Re}_x^{\frac{1}{n+1}}}{\text{Ux}r_0} \right) \psi(x, r), \]

\[ \theta(\xi, \eta) = \frac{T - T_\infty}{T_\infty - T_\infty}, \quad \phi(\xi, \eta) = C - C_\infty \quad u = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad \text{and} \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial r}. \]

Where \( \psi(x, r) \) is the stream function \( \text{Re}_x = \rho U^{2-n} x^n K^{-1} \) is the local generalized Reynolds number.

Introducing the previous new variables into equations (4)-(7) we obtain

\[ \frac{\partial}{\partial \eta} \left[ (1 + \alpha \eta)^{\frac{n+1}{2}} f'' + \frac{1}{n+1} f f'' - \frac{M}{\alpha_\varepsilon^2 + \beta_e^2} (\alpha_\varepsilon f' + \beta_e g) \right] = \xi f' \frac{\partial f'}{\partial \xi} - f' \frac{\partial f}{\partial \xi} \]

\[ \frac{\partial}{\partial \eta} \left[ (1 + \alpha \eta)^{\frac{n+1}{2}} g'' + \frac{1}{n+1} f g' + \frac{M}{\alpha_\varepsilon^2 + \beta_e^2} (\beta_e f' - \alpha_\varepsilon g) \right] = \xi f' \frac{\partial g'}{\partial \xi} - g' \frac{\partial f}{\partial \xi} \]

\[ \frac{1}{\text{Pr}} \frac{\xi^{\frac{n+1}{(n+1)}}}{\partial \eta} \left[ (1 + \alpha \eta) \theta' \right] + \frac{1}{n+1} f \theta' - \lambda f' \theta + M Ec \left( f'^2 + g^2 \right) = \xi f' \frac{\partial \theta}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \]

\[ \frac{1}{\text{Sc}} \frac{\xi^{\frac{n+1}{(n+1)}}}{\partial \eta} \left[ (1 + \alpha \eta) \phi' \right] + \frac{1}{n+1} f \phi' - mf' \phi + \delta \phi' = \xi f' \frac{\partial \phi}{\partial \xi} - \phi \frac{\partial f}{\partial \xi} \]

The boundary conditions (8) are transformed to:

\[ f'(\xi, 0) = 1, \quad f(\xi, 0) = (n + 1) \xi \frac{\partial f}{\partial \xi} \bigg|_{\eta=0} = 0, \quad g(\xi, 0) = 0 \quad \theta(\xi, 0) = \phi(\xi, 0) = 1 \]

And \( f'(\xi, \infty) = g(\xi, \infty) = \theta(\xi, \infty) = \phi(\xi, \infty) = 0 \)

Where primes in the above equations designate differentiation with respect to \( \eta \) only.

The governing boundary layer equations (9–12) subject to the boundary conditions (13) are approximated to a system of non–linear ordinary differential equation by replacing the derivatives with respect to \( \xi \) by two–point back ward finite differences. These equations are integrated by the fourth order Runge–Kutta method with a modified version of Newton–Raphson shooting method with step size 0.05.
The physical quantities of fundamental interest of heat and mass transfer study are skin friction components $\tau_{wx}$, $\tau_{zw}$, the local Nusselt number $Nu_x$ and the local Sherwood number $Sh_x$.

In view of the velocity field, the local skin friction components are given by:

$$
\tau_{wx} = K \left( \frac{\partial u}{\partial r} \right) \left|_{r=r_0} \right. = K \left( \frac{U}{x} \right)^n \Re_x^{\frac{n}{n+1}} (f''(\xi,0))^n
$$

(14)

$$
\tau_{zw} = K \left( \frac{\partial w}{\partial r} \right) \left|_{r=r_0} \right. = K \left( \frac{U}{x} \right)^n \Re_x^{\frac{n}{n+1}} (g'(\xi,0))^n
$$

(15)

The local Nusselt number is given by:

$$
Nu_x = -\Re_x^{\frac{1}{n(n+1)}} \theta'(\xi,0)
$$

(16)

Also, the local Sherwood number is given by:

$$
Sh_x = -\Re_x^{\frac{1}{n(n+1)}} \phi'(\xi,0)
$$

(17)

3. Results and Discussion

The numerical solutions were conducted for $\xi = 0.05$, $Pr = 10$, $Sc = 0.5$, $\varepsilon = 0.5$ and various values of the parameters entering into the problem, namely, the power–law viscosity index $n$, the magnetic parameter $M$, the reaction rate parameter $\delta$, the order of the chemical reaction $\gamma$, the Eckert number $Ec$, the Hall parameter $\beta_e$, the ion-slip parameter $\beta_i$, and the exponents $m$ and $\lambda$. The effects of the previous mentioned parameters were examined on the flow of a shear-thinning or pseudo-plastic fluid with $n = 0.5$. The chemical reaction is assumed to be destructive so $\delta$ is taken to be positive (i.e., $\delta > 0$). The effect of the power-law viscosity index $n$ is studied for a shear-thinning fluids ($n < 1$), Newtonian fluids ($n = 1$) and shear-thickening fluids ($n > 1$). It is obvious from Figures 1-4 that the increase of $n$ has a tendency to decrease $f'(\xi, \eta)$, $\theta$ and $\phi(\xi, \eta)$ and to increase the secondary velocity $g(\xi, \eta)$ as shown in figures 5-7. The effect of the magnetic field on the concentration profiles is found to be insensible so no figure of these variables is presented herein. This behavior due to the resistive force which has a tendency to slow down the fluid primary velocity in the boundary layer, while the increase in the secondary flow velocity due to the positive body force in the transformed momentum equation (10). The temperature distribution $\theta(\xi, \eta)$ decreases owing to the increase in the temperature power coefficient $\lambda$, while it increases as the Eckert number increases. This behavior is shown in Figures 8 and 9 respectively. However, as $\lambda$ becomes small ($\lambda = -1$) and as the Eckert number gets large ($Ec = 50$) a distinctive peak in
the temperature distribution $\theta(\xi, \eta)$ occurs in the fluid adjacent to the cylinder surface. This means that the temperature of the fluid in the boundary layer is higher than the surface temperature, so heat is expected to transfer to the cylinder's surface. Figures 10, 11 and 12 depict the influence of the reaction order $\gamma$, the reaction rate parameter $\delta$ and the wall concentration coefficient $m$ on the concentration distribution $\phi(\xi, \eta)$, respectively. It is obvious that the concentration $\phi(\xi, \eta)$ increases as $\gamma$ increases, while it decreases owing to the increase in $\delta$ or $m$. The increase of the diffusing species absorption yields the concentration boundary layer to become thinner and the concentration of the diffusing species to decrease. This is true since $0 \leq \phi(\xi, \eta) \leq 1$ and the reaction is destructive. The results incorporate the effect of the Hall parameter $\beta_e$ on $g(\xi, \eta)$ profiles is presented in Figure 13. We observe that the velocity profiles $g(\xi, \eta)$ decrease with increasing the Hall parameter $\beta_e$. On the other hand, the effect of the Hall parameter $\beta_e$ on the temperature and concentration profiles is found to be insensible. Figure 14 shows as expected that the velocity $g(\xi, \eta)$ decreases with increasing the ion-slip parameter $\beta_i$, while the increase of $\beta_i$ slightly increases the temperature and concentration profiles. For the sake of brevity, no figure of these variables is presented herein. The evolution of $\xi$ on the temperature $\theta((\xi, \eta)$ and concentration $\phi(\xi, \eta)$ for the case of a shear thinning or pseudo-plastic fluids with $n = 0.5$ is illustrated in Figures 15 and 16. It is obvious that the increase of $\xi$ decreases both the temperature and concentration profiles. On the other hand, in the case of shear-thickening fluids the increase of $\xi$ increases both the temperature and concentration profiles, as illustrated in Figures 17 and 18.

Numerical values of $f''(\xi, 0)$, $g'(\xi, 0)$, $\theta'(\xi, 0)$ and $\phi'(\xi, 0)$ for $Pr = 10$, $Sc = 5$ and $\epsilon = 0.5$ are presented in tables. Table 1 shows the variation of $\theta'(\xi, 0)$, which represents the local Nusselt number and $\phi'(\xi, 0)$, which represents the local Sherwood number for $\xi = 0.05$, $M = 0.1$, $n = 0.5$, $\beta_e = \beta_i = 0.5$ and various values of $\delta$, $\gamma$, $m$, $\lambda$ and $Ec$. It is clear that the local Nusselt number increases as $\lambda$ increases, while the increase of the Eckert number has the opposite effect on $-\theta'(\xi, 0)$. Also, it is noted that $\phi'(\xi, 0)$ decreases as $\gamma$ increases, while it increases as $\delta$ or $m$ increases. Table 2 presents numerical values of $f''(\xi, 0)$, $g'(\xi, 0)$ which represent the two components of the skin friction for $\xi = 0.05$, $M = 0.1$, $\lambda = \gamma = \delta = m = 1$, $Ec = 0.1$, $n = 0.5$ and various values of $\beta_e$ and $\beta_i$. It is clear that the increase of $\beta_e$ or $\beta_i$ decreases both $f''(\xi, 0)$ and $g'(\xi, 0)$. Table 3 presents the numerical values of $f''(\xi, 0)$, $g'(\xi, 0)$, $\theta'(\xi, 0)$ and $\phi'(\xi, 0)$ for $\gamma = \lambda = \delta = m = 1$, $Ec = 0.1$, and various values of the magnetic parameter $M$, the power-law viscosity index $n$ and $\xi$. It is obvious from the table that the increase of $n$ has a tendency to increase $g'(\xi, 0)$, $\theta'(\xi, 0)$ and $\phi'(\xi, 0)$ and to decrease $f''(\xi, 0)$, while the increase of $M$ increases both $f''(\xi, 0)$ and $g'(\xi, 0)$ and decreases $\theta'(\xi, 0)$. The local Sherwood number decreases as $M < 0.5$, increases as $M > 0.5$ and attains its minimum value at $M = 0.5$. Moreover, it is clear that the local Nusselt and Sherwood numbers increase for pseudo-plastic fluids ($n < 1$) and decrease for dilatants fluids ($n > 1$).

4. Concluding remarks

In this paper, we have studied the combined effect of the Ohmic heating and the Hall and ion-slip currents on the flow of non-Newtonian fluids over continuously moving cylinder using the non-similarity approach. The flow is subjected to a uniform strong magnetic field. In
this analysis we have focused on the case of power-low variation in the temperature and concentration at the cylinder surface. The governing boundary conditions are transformed to a non-similar form by introducing appropriate transformation. The transformed equations have been solved numerically using the Runge–Kutta method with a modified version of Newton-Raphson shooting technique.

The present investigation has shown that

1) The primary flow velocity \( f'(\xi, \eta) \) decreases as \( M \) and \( n \) increase. Moreover, the skin friction component \( \tau_{wx} \) increases as \( M \) increases, while it decreases as any of \( \beta_e \) or \( \beta_i \) increases.

2) The increase of the Hall and ion-slip parameters have the tendency to decrease the secondary flow velocity \( g(\xi, \eta) \) and the skin friction component \( \tau_{wz} \), while the increase of \( M \) and \( n \) yields to increase the skin friction component \( \tau_{wz} \).

3) The temperature distribution \( \theta(\xi, \eta) \) increases and the local Nusselt number decreases as any of the Ec, \( M \) and \( \xi \) in the case of \( n = 1.5 \) increases, while the effect occurs as \( n, \lambda \) or \( \xi \) in the case of \( n = 0.5 \) increases.

4) The increase of \( \delta, m, n \) and \( \xi \) in the case of \( n = 0.5 \) decrease the concentration \( \phi(\xi, \eta) \) and increase the local Sherwood number. On the other hand, the increase of \( \gamma \) and \( \xi \) in the case of \( n = 1.5 \) increase the concentration distribution and decrease the local Sherwood number.
Fig. 5. Variation of $f(\xi, \eta)$ for $\xi = 0.05, \delta = \lambda = \gamma = m = 1, Ec = 0.1, n = \beta_c = \beta_i = 0.5$

Fig. 6. Variation of $g(\xi, \eta)$ for $\xi = 0.05, \delta = \lambda = \gamma = m = 1, Ec = 0.1, n = \beta_c = \beta_i = 0.5$

Fig. 7. Variation of $\theta(\xi, \eta)$ for $\xi = 0.05, \delta = \lambda = \gamma = m = 1, Ec = 0.1, n = \beta_c = \beta_i = 0.5$

Fig. 8. Variation of $\theta(\xi, \eta)$ for $\xi = 0.05, \delta = \gamma = m = 1, M = Ec = 0.1, n = \beta_c = \beta_i = 0.5$

Fig. 9. Variation of $\theta(\xi, \eta)$ for $\xi = 0.05, \delta = \lambda = \gamma = m = 1, M = 0.1, n = \beta_c = \beta_i = 0.5$

Fig. 10. Variation of $\phi(\xi, \eta)$ for $\xi = 0.05, \delta = \lambda = \gamma = m = 1, Ec = M = 0.1, n = \beta_c = \beta_i = 0.5$

Fig. 11. Variation of $\phi(\xi, \eta)$ for $\xi = 0.05, \lambda = \gamma = m = 1, Ec = M = 0.1, n = \beta_c = \beta_i = 0.5$

Fig. 12. Variation of $\phi(\xi, \eta)$ for $\xi = 0.05, \lambda = \gamma = \delta = 1, Ec = M = 0.1, n = \beta_c = \beta_i = 0.5$
Fig. 13. Variation of $g(\xi, \eta)$ for $\xi = 0.05, \lambda = \gamma = m = \delta = 1, Ec = M = 0.1, n = \beta_e = 0.5$

Fig. 14. Variation of $g(\xi, \eta)$ for $\xi = 0.05, \lambda = \gamma = m = \delta = 1, Ec = M = 0.1, n = \beta_i = 0.5$

Fig. 15. Variation of $\theta(\xi, \eta)$ for $\lambda = \gamma = m = \delta = 1, Ec = M = 0.1, \beta_e = \beta_i = 0.5$

Fig. 16. Variation of $\phi(\xi, \eta)$ for $\lambda = \gamma = m = \delta = 1, Ec = M = 0.1, \beta_e = \beta_i = 0.5$

Fig. 17. Variation of $\theta(\xi, \eta)$ for $\lambda = \gamma = m = \delta = 1, Ec = M = 0.1, \beta_e = \beta_i = 0.5$

Fig. 18. Variation of $\phi(\xi, \eta)$ for $\lambda = \gamma = m = \delta = 1, Ec = M = 0.1, \beta_e = \beta_i = 0.5$
Table 1: Variation of $-\phi''(\xi, 0)$ and $-\theta'(\xi, 0)$ for $\xi = 0.05$, $M = 0.1$, $\beta_e = \beta_i = 0.5$ and $n = 0.5$

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Table 2: Variation of $f''(\xi, 0)$ and $g'(\xi, 0)$ for $\xi = 0.05$, Ec = M =0.1, $\lambda = \gamma = \delta = m = 1$ and $n = 0.5$

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Table 3: Variation of \( f''(\xi, 0) \), \( g'(\xi, 0) \), \( \theta'(\xi, 0) \) and \( \phi'(\xi, 0) \) for \( Ec = 0.1, \lambda = \gamma = \delta = m = 1, \beta_e = \beta_1 = 0.5 \)

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References


Sakiadis, B. C. (1961), Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow, AICHE journal, 7, 467.
